Theory of magnetohydrodynamic and shock waves in neutron matter

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The magnetohydrodynamic equations for a nonconducting magnetically ordered medium are developed. Using these equations, we investigate magnetic neutron matter as a magnetically ordered nonconducting liquid. It is shown that in this media small amplitude waves such as modified spin and sound waves and shock waves may propagate. The differential conservation laws for the densities of additive integrals of motion are constructed. On this basis we find the relations between the discontinuities of hydrodynamic quantities for the shock. The extension of the Hugoniot adiabat for shock waves in magnetic neutron matter is obtained.
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I. INTRODUCTION

As was previously shown [1], a phase transition from normal to magnetically ordered states can take place in dense neutron matter. In this matter the propagation of specific magnetohydrodynamic waves of small amplitude, as well as the propagation of shock waves, are possible.

Note that in studying these waves one cannot directly employ the equations of standard magnetohydrodynamics, because these assume electrical media. However, neutron matter is nonconductive. Therefore, we have to obtain new equations to replace the standard magnetohydrodynamics equations.

In this paper we establish magnetohydrodynamic equations for a nonconducting medium and investigate the small and large amplitude waves in magnetic neutron matter. In Sec. II the magnetohydrodynamics of nonconducting media is developed, and the corresponding equations are formulated. Section III is devoted to an investigation of magnetohydrodynamic waves of small amplitude, and to obtaining their spectra. The theory of a shock waves for neutron matter is developed in Sec. IV. We work in a nonrelativistic framework.

II. DYNAMIC EQUATIONS FOR NONCONDUCTING MAGNETICALLY ORDERED MEDIUM

We derive dynamic equations for a medium where magnetodipole interaction between neutrons plays an important role. Let us suppose that the medium is described by the displacement vector $\mathbf{u}(\mathbf{x},t)$, density $\rho(\mathbf{x},t)$, density of magnetic moment $\mathbf{M}(\mathbf{x},t)$, and entropy density $\sigma(\mathbf{x},t)$. Note that the position x_i of a medium particle at the moment t is the function of its initial coordinates ξ_i, $x_i=x_i(\xi,t)$ (ξ_i are the Lagrange coordinates). If we introduce the displacement vector $\mathbf{u}(\xi,t)$,

$$x_i(\xi,t)=\xi_i+u_i(\xi,t), \tag{2.1}$$

then we can express ξ as $\xi_i=\xi_i(\mathbf{x},t)$ ($\mathbf{x}$ are the Euler coordinates). Further, let us consider the displacement vector not as a function of ξ and t but as a function of $\mathbf{x}$ and t, $u_i=u_i(\mathbf{x},t)$. Thus the velocity of a medium particle is defined by the equation

$$v_i(\mathbf{x},t)=\frac{\partial x_i(\xi,t)}{\partial t}=\frac{\partial u_i(\mathbf{x},t)}{\partial t}+v_j(\mathbf{x},t)\,\frac{\partial u_i(\mathbf{x},t)}{\partial x_j}. \tag{2.2}$$

So we have

$$\frac{\partial u_i(\mathbf{x},t)}{\partial t}=b_{ij}v_j(\mathbf{x},t), \tag{2.3}$$

where $b_{ij}=\delta_{ij}-\partial u_i/\partial x_j\equiv\partial\xi_i/\partial x_j$. It easy to see that the medium density may be represented as

$$\rho(\mathbf{x},t)=\rho_0\,\det\frac{\partial\xi_i}{\partial x_k}, \tag{2.4}$$

where ρ_0 is the density of a nondeformed medium.

We immediately obtain, from Eqs. (2.3) and (2.4),

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\,\rho\mathbf{v}=0. \tag{2.5}$$

We write the equation of motion of the magnetic moment density in the form

$$\frac{\partial M_i}{\partial t}+\operatorname{div}\,M_i\mathbf{v}=g[\mathbf{M},\mathbf{H}_{\text{eff}}]_i, \tag{2.6}$$

where g is the gyromagnetic ratio and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field that acts on the magnetic moment of the medium.

We write the medium dynamic equation as

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t}+v_i(\mathbf{x},t)\,\frac{\partial\mathbf{v}}{\partial x_i}\right)=\mathbf{f}, \tag{2.7}$$

where $\mathbf{f}$ is the force that acts on unit volume of the medium.

For the entropy density we have an equation

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$$\frac{\partial \sigma}{\partial t} + \text{div } \sigma \mathbf{v} = 0. \tag{2.8}$$

The last equation is usually called the adiabaticity equation.

As long as we do not take into consideration the dissipative processes, the effective magnetic field $\mathbf{H}_{\text{eff}}$ and force $\mathbf{f}$ are the functionals of $\mathbf{u}$, $\mathbf{M}$, and σ, and do not depend on velocity $\mathbf{v}$. Let us clarify the structure of the functionals in terms of the phenomenological energy W defined by equation

$$W = \int d^3x \tfrac{1}{2} \rho v^2 + W_0(\mathbf{M}, \mathbf{u}, \sigma), \tag{2.9}$$

where $\frac{1}{2}\rho v^2$ is the density of the medium kinetic energy, and the energy W_0 consists of the energy of short-range interactions and the energy of long-range magnetodipole interaction. Differentiating Eq. (2.9) with respect to t and taking into account Eqs. (2.3) and (2.5)–(2.8), we obtain

$$\frac{\partial W}{\partial t} = \int d^3x \; v_i \left(f_i + \frac{\delta W_0}{\delta u_j} b_{ji} + \sigma \frac{\partial}{\partial x_i} \frac{\delta W_0}{\delta \sigma} + M_j \frac{\partial}{\partial x_i} \frac{\delta W_0}{\delta M_j} \right) + \int d^3x \frac{\delta W_0}{\delta M_j} g[\mathbf{M}, \mathbf{H}_{\text{eff}}]_j . \tag{2.10}$$

Due to the energy conservation law the derivative $\partial W / \partial t$ must be zero at arbitrary $\mathbf{v}$, $\mathbf{u}$, and $\mathbf{M}$. Substituting $\mathbf{v}=0$ in Eq. (2.10), one can easily see that $\mathbf{H}_{\text{eff}}$ should be identified with the quantity $-\delta W_0 / \delta \mathbf{M}$:

$$\mathbf{H}_{\text{eff}} = -\frac{\delta W_0}{\delta \mathbf{M}}. \tag{2.11}$$

Then $\partial W / \partial t$ will go to zero at arbitrary $\mathbf{v}$ if

$$f_i = \frac{\delta W_0}{\delta u_j} b_{ji} - \sigma \frac{\partial}{\partial x_i} \frac{\delta W_0}{\delta \sigma} - M_j \frac{\partial}{\partial x_i} \frac{\delta W_0}{\delta M_j}. \tag{2.12}$$

The energy W_0 is not a functional of the displacement vector $\mathbf{u}$ but of $[\partial u_i(\mathbf{x},t)]/\partial x_j$ or the quantity b_{ij}. So we have

$$\frac{\delta W_0}{\delta u_i(\mathbf{x})} = \frac{\partial}{\partial x_k} \frac{\delta W_0}{\delta b_{ik}(\mathbf{x})}. \tag{2.13}$$

Equations (2.5)–(2.8), (2.11), and (2.12) describe the dynamics of a nonconductive magnetic medium. In the case of small spatial gradients these equations transform into Eqs. (15.1.1), (15.1.8), (15.1.9) of Ref. [2].

Let us clarify the term of the magnetodipole interaction in W_0 in Eq. (2.9). With this purpose we note that for a nonconductive medium the electric and magnetic fields satisfy the equations

$$\text{curl}\mathbf{H} = \mathbf{0}, \quad \text{div } \mathbf{B} = \mathbf{0},$$

$$\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M},$$

$$-\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = \text{curl}\mathbf{E}. \tag{2.14}$$

Then the energy W_0 may be represented by

$$W_0(\mathbf{M}, b, \sigma) = \int d^3x \left\{ \frac{1}{2} \alpha_{ij} \frac{\partial \mathbf{M}}{\partial x_i} \cdot \frac{\partial \mathbf{M}}{\partial x_j} + w(M^2, b, \sigma) - \frac{1}{2} \mathbf{M} \cdot \mathbf{H} \right\}, \tag{2.15}$$

where the first term is the exchange energy due to inhomogeneity of the magnetic moment, α is the exchange constant, $w(M^2,b,\sigma)$ is the energy density of the homogeneous state due to short-range forces, and the last term is the energy of the magnetodipole interaction, which may be rewritten as follows:

$$\int d^3x \frac{\mathbf{H}^2}{8\pi} = -\frac{1}{2} \int d^3x \; \mathbf{M} \cdot \mathbf{H}.$$

The system of equations (2.5)–(2.8), (2.11), and (2.12) allows one to describe both the magnetic dielectrics and the paramagnetic liquids. For liquid the energy density depends on b_{ij} only by means of density of liquid $\rho(\mathbf{x},t) = \rho_0 \det b$; moreover, $\alpha_{ij} = \alpha \delta_{ij}$, $w = w(\mathbf{M}^2, \rho, \sigma)$. Noting that

$$b_{ij} \frac{\partial \det b}{\partial b_{jk}} = (\det b) \delta_{ij},$$

let us represent the force $\mathbf{f}$ and the effective magnetic field $\mathbf{H}_{\text{eff}}$ as

$$f_i = \mathbf{M} \cdot \frac{\partial \mathbf{H}}{\partial x_i} - \frac{\partial P}{\partial x_i}, \tag{2.16}$$

$$\mathbf{H}_{\text{eff}} = \mathbf{H} + \alpha_{ij} \frac{\partial^2 \mathbf{M}}{\partial x_i \partial x_j} - \frac{\partial}{\partial \mathbf{M}} \int d^3x \; w(M^2, b, \sigma), \tag{2.17}$$

where

$$P = \rho \frac{\partial w}{\partial \rho} + \sigma \frac{\partial w}{\partial \sigma} + M_i \frac{\partial w}{\partial M_i} - w \tag{2.18}$$

is the liquid pressure.

Equation (2.18) may be obtained from the thermodynamics consideration also. As is well known, the entropy σ and Gibbs potential ω are related by the equation

$$\sigma = -\omega + \frac{w}{T} + \frac{\mu}{T} \rho + \frac{\mathbf{h}}{T} \cdot \mathbf{M}, \tag{2.19}$$

where T is the temperature, μ is the chemical potential, and $\mathbf{h}$ is the effective magnetic field in the spatially homogeneous case. Since the Gibbs potential ω satisfies the thermodynamical identity

$$d\omega = w d \frac{1}{T} + \rho d \frac{\mu}{T} + \mathbf{M} \cdot d \frac{\mathbf{h}}{T}, \tag{2.20}$$

the quantity dw can be written as

$$dw = T d\sigma - \mu d\rho - \mathbf{h} \cdot d\mathbf{M}. \tag{2.21}$$

Using the definition $P=-T\omega$ and Eq. (2.21), we again obtain Eq. (2.18). Equations (2.16)–(2.18) describe the dynamics of both magnetically ordered and nonmagnetic neutron matter.

III. MAGNETOHYDRODYNAMIC WAVES IN DENSE NEUTRON MATTER

Let us apply the set of equations (2.5)–(2.8), (2.11), and (2.12) to an investigation of small amplitude magnetohydrodynamic waves. We suppose that the equilibrium state, small deviations from which we study, is characterized by the equilibrium density ρ_0 and spontaneous magnetization $\mathbf{M}_0$, whereas the equilibrium velocity and the external magnetic field equal zero, $\mathbf{v}_0=0$ and $\mathbf{H}=0$. Linearizing Eqs. (2.5)–(2.8), (2.11), and (2.12) near this equilibrium state, we come to the dispersion relation

$$\omega^4-A\omega^2+B=0, \tag{3.1}$$

where

$$A=\omega_s^2+\omega_0^2+\omega_1^2(\cos^2\theta+\alpha\rho_0 g^2\sin^2\theta)>0, \tag{3.2}$$

$$B=\omega_s^2(\omega_0^2+\omega_1^2\cos^2\theta)+\omega_1^2\omega_0^2\alpha\rho_0 g^2\sin^2\theta>0. \tag{3.3}$$

Here θ is the angle between the wave vector $\mathbf{k}$ and the magnetization $\mathbf{M}_0$, $\omega_s=gM_0\alpha k^2$ is the spin wave frequency, $\omega_0=k\sqrt{\partial P/\partial\rho}|_{\bar{\sigma},\bar{M}}$ is the acoustic wave frequency, $\bar{\sigma}$ is the unit mass entropy, M is the unit mass magnetic moment, P is the pressure of the neutron matter, and $\omega_1^2=4\pi\rho_0^{-1}M_0^2k^2$. From Eq. (3.1) we obtain the frequencies of magnetohydrodynamic waves

$$\omega_\pm^2=\frac{A\pm\sqrt{A^2-4B}}{2}. \tag{3.4}$$

It is easy to verify that the following relation results from the Eqs. (3.2), (3.3)

$$A^2-4B=\{\omega_s^2-\omega_0^2+\omega_1^2(\alpha\rho_0 g^2\sin^2\theta-\cos^2\theta)\}^2+\omega_1^4\alpha\rho_0 g^2\sin^2 2\theta>0;$$

therefore, $\omega_\pm^2>0$. Let us analyze the dispersion relation (3.4) in the limit of small and large wave vectors k. At small k, the term $\omega_s^2\sim k^4$ can be neglected in the expression for A^2 and A^2-4B. Then in the case of weak magnetoelastic coupling ($\omega_1^2\ll\omega_0^2$ or $4\pi M_0^2/\rho_0\ll\partial P/\partial\rho$) we have

$$\omega_+^2=\omega_0^2+\omega_1^2\cos^2\theta,\quad \omega_-^2=\omega_1^2\alpha\rho_0 g^2\sin^2\theta. \tag{3.5}$$

The branch ω_+ is seen to be a slightly modified ordinary acoustic wave, while ω_- in the region of small k coincides with the spin branch modified by dipole-dipole interaction. In the opposite case of strong magnetization ($\omega_1^2\gg\omega_0^2$, or $4\pi M_0^2/\rho_0\gg\partial P/\partial\rho$), we have

$$\omega_+^2=\omega_1^2(\cos^2\theta+\alpha\rho_0 g^2\sin^2\theta)+\omega_0^2\,\frac{\cos^2\theta}{\cos^2\theta+\alpha\rho_0 g^2\sin^2\theta},$$

$$\omega_-^2=\omega_0^2\,\frac{\rho_0\alpha g^2\sin^2\theta}{\cos^2\theta+\alpha\rho_0 g^2\sin^2\theta}.$$

In the limit of the large wave vectors $k\to\infty$, when

$$A^2-4B=\omega_s^4-2\omega_s^2[\omega_0^2-\omega_1^2(\alpha\rho_0 g^2\sin^2\theta-\cos^2\theta)],$$

we have

$$\omega_+^2=\omega_s^2+\omega_1^2\alpha\rho_0 g^2\sin^2\theta,\quad \omega_-^2=\omega_0^2+\omega_1^2\cos^2\theta.$$

The branch ω_+ exactly coincides with the well-known expression for the frequency of the spin wave modified by dipole-dipole interaction. In the case of strong magnetization the branch ω_-^2 is given by

$$\omega_-^2=4\pi\,\frac{M_0^2}{\rho_0}\,k^2\cos\theta.$$

This branch is similar to the standard magnetohydrodynamic Alfven wave [3] $\omega_-^2=(H^2/4\pi\rho_0)k^2\cos\theta$, where H is the external magnetic field.

IV. SHOCK WAVES

We have applied Eqs. (2.5)–(2.8), (2.11), and (2.12) for an investigation of small amplitude magnetohydrodynamic waves in neutron matter. These equations allow one to consider the shock waves in neutron matter. To investigate shocked flows it is convenient to transform Eqs. (2.5)–(2.8) into the form of differential conservation laws for the particle density ρ, momentum density $\rho\mathbf{v}$, energy density w, and angular momentum density. For simplicity we take into account only magnetodipole interaction, and neglect the exchange interaction ($\alpha=0$). Then the equations for ρ and ρv_i are given by

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_i}\rho v_i=0, \tag{4.1}$$

$$\frac{\partial\rho v_i}{\partial t}+\frac{\partial}{\partial x_k}\Pi_{ik}=0, \tag{4.2}$$

where the density of moment flow Π_{ik} is defined as

$$\Pi_{ik}=\rho v_i v_k+P\delta_{ik}-\frac{1}{4\pi}\left(H_iB_k-\frac{1}{2}\mathbf{H}^2\delta_{ik}\right). \tag{4.3}$$

The equation of adiabaticity of flow is transformed to a differential law of energy conservation,

$$\frac{\partial w}{\partial t}+\frac{\partial}{\partial x_i}\Pi_i=0, \tag{4.4}$$

where the energy density w and the density of energy flow Π_k are defined by

$$w=\frac{1}{2}\rho v^2+w_0(M^2,\rho,\sigma)+\frac{\mathbf{H}^2}{8\pi},$$

$$\Pi_k=\tfrac{1}{2}\rho v^2 v_k+\frac{c}{4\pi}[\mathbf{E},\mathbf{H}]_k-\mathbf{H}\cdot\mathbf{M}v_k+v_k(P+w_0). \tag{4.5}$$

Taking into account Eqs. (4.2) and (4.3), we can transform Eq. (2.6) into the differential conservation law for angular momentum,

$$\frac{\partial}{\partial t}\left([\mathbf{r},\rho\mathbf{v}]+\frac{1}{g}\,\mathbf{M}\right)_i+\frac{\partial}{\partial x_k}\,J_{ik}=0, \tag{4.6}$$

where J_{ik} is defined by

$$J_{ik}=\varepsilon_{isl}x_s\Pi_{lk}+\frac{1}{g}\,M_i v_k\,.$$

As a first step let us consider one-dimensional flow. In other words, the physical quantities that describe the flow depend on one coordinate x. We denote by $a_\alpha(x)$ the densities of physical quantities $(a_\alpha(x)=\rho(x),\ \rho(x)v_i(x),\ [\mathbf{r},\rho\mathbf{v}]_i+(1/g)\mathbf{M}_i)$ and by $b_{\alpha k}(x)$ the densities of corresponding flows, which are the functions of $a_\alpha(x)$ $[b_{\alpha k}(x)=b_{\alpha k}(a_\beta(x))]$. Then the differential conservation laws, Eqs. (4.1), (4.2), (4.4), and (4.6), may be expressed as

$$\dot{a}_\alpha+\frac{\partial b_{\alpha k}}{\partial x_k}=0,$$

or, in the one-dimensional case,

$$\dot{a}_\alpha+\frac{\partial b_\alpha}{\partial x}=0 \quad (b_\alpha(x)=b_\alpha a_\beta(x)), \tag{4.7}$$

where $b_\alpha=b_{\alpha x}$.

To describe the shocked flows we assume that

$$a_\alpha(x)=a_\alpha^+(x)\,\theta(x-x(t))+a_\alpha^-(x)\,\theta(-x+x(t)), \tag{4.8}$$

where $x(t)$ is the x coordinate of the shock front at the moment t,

$$\theta(x)=\begin{cases}1, & x\geqslant 0\\ 0, & x<0.\end{cases}$$

Therefore, b_α is given by

$$b_\alpha(x)=b_\alpha^+(x)\,\theta(x-x(t))+b_\alpha^-(x)\,\theta(-x+x(t)), \tag{4.9}$$

where $b_\alpha^\pm(x)=b_\alpha(a_\alpha^\pm)$.

Substituting the Eqs. (4.8) and (4.9) into Eq. (4.7), and taking into account that $(\partial/\partial t)\,\theta(x-x(t))=-\dot{x}(t)\,\delta(x-x(t))$ we obtain

$$\left(\dot{a}_\alpha^+ +\frac{\partial b_\alpha^+}{\partial x}\right)\theta(x-x(t))+\left(a_\alpha^- +\frac{\partial b_\alpha^-}{\partial x}\right)\theta(-x+x(t))$$
$$+\,\delta(x-x(t))(-\dot{x}(t))(a_\alpha^+-a_\alpha^-)+(b_\alpha^+-b_\alpha^-)=0. \tag{4.10}$$

Thus we find the magnetohydrodynamic equations in front of shock and behind the shock,

$$\dot{a}_\alpha^\pm+\frac{\partial b_\alpha^\pm}{\partial x}=0, \tag{4.11}$$

and the boundary conditions at the shock surface,

$$\{b_\alpha\}=\dot{x}(t)\{a_\alpha\}, \tag{4.12}$$

where {} denotes the difference between physical quantities in front of and behind the shock. For example,

$$\{a_\alpha\}=(a_\alpha^+-a_\alpha^-)|_{x=x(t)}\,.$$

Using Maxwell equations in a form similar to Eq. (4.11), we find the boundary conditions for electromagnetic fields at the shock surface

$$\{\epsilon_{i1k}E_k\}=\frac{\dot{x}(t)}{c}\,\{B_i\},\quad \{\epsilon_{i1k}H_k\}=0,\quad \{B_1\}=0.$$

It is easy to rewrite these conditions as

$$\left\{\left(\mathbf{E}+\frac{1}{c}\,[\dot{\mathbf{x}},\mathbf{B}]\right)\right\}_t=\mathbf{0},\quad \{\mathbf{H}_t\}=\mathbf{0},\quad \{\mathbf{B}_n\}=\mathbf{0}, \tag{4.13}$$

where n denotes the electromagnetic fields component normal to the shock front, and t denotes the components along the shock front. Note that $\dot{\mathbf{x}}=\mathbf{u}(t)$ is the normal component of the shock velocity. The conditions in the form of Eqs. (4.12) take place for any flow (not necessary one dimensional). In a similar way, Eqs. (4.12) may be written for some arbitrary shocked flow

$$\{b_{\alpha k}n_k\}=u(t)\{a_\alpha\},\quad u(t)=|\mathbf{u}(t)|.$$

In the case of a plane shock wave moving with the constant velocity $\mathbf{u}$, the magnetohydrodynamic equations have the solutions

$$a_\alpha^+=\text{const},\quad a_\alpha^-=\text{const},$$

the above constants being related by

$$b_\alpha(a^+)-b_\alpha(a^-)=u(a_\alpha^+-a_\alpha^-). \tag{4.14}$$

Equation (4.14) is valid for an arbitrary rest frame. We consider this equation in the frame where the shock front is rested, $\mathbf{u}=0$. So we have

$$\{\rho\mathbf{v}\cdot\mathbf{n}\}=0,\quad \{\pi_{ik}n_k\}=0,$$
$$\{\Pi_k n_k\}=0,\quad \{J_{ik}n_k\}=0, \tag{4.15}$$

where $\mathbf{n}$ is the unit normal vector of the shock front. Equation (4.14) is used in the standard hydrodynamics and magnetohydrodynamics of shock waves (see, for example, Refs. [3,4]).

Equations (4.15) must be supplemented by the equations for the discontinuities of electromagnetic fields,

$$\{B_k n_k\}=0,\quad \varepsilon_{ikl}\{H_k n_l\}=0,\quad \varepsilon_{ikl}\{E_k n_l\}=0. \tag{4.16}$$

Using the definition of densities of flows (4.5) and (4.6), let us rewrite Eqs. (4.15) and (4.16) as follows

$$\left\{\rho v_n\left(\frac{v_n^2}{2}+\frac{v_t^2}{2}\right)+\rho v_n\left(\frac{P+w_0}{\rho}\right)-\mathbf{H}\cdot\mathbf{M}v_n\right\}=0,$$

$$\left\{P+\rho v_n^2-\frac{H_n^2}{8\pi}-H_nM_n\right\}=0,$$

$$\left\{\rho v_n v_t-\frac{1}{4\pi}\,\mathbf{H}_t B_n\right\}=0,\quad \{v_n\mathbf{M}\}=0. \tag{4.17}$$

Here we use the condition $\{H_t^2\}=0$. Taking into account the first equation from Eqs. (4.16), the third equation from Eqs. (4.17) may be rewritten as $\{\rho v_n v_t\}=0$.

The shock waves are waves where the density ρ is discontinuous at the shock front [3]. Denoting by j the normal component of the matter density (it is continuous at the shock front, $j\neq 0$) and introducing $V=1/\rho$ (V is the specific volume) we can rewrite Eqs. (4.17) in the forms

$$\left\{(P+w_0)V+\frac{j^2V^2}{2}-V\mathbf{H}\cdot\mathbf{M}\right\}=0,$$

$$\left\{P+j^2V-\frac{H_n^2}{8\pi}-H_nM_n\right\}=0,$$

$$\{v_t\}=0,\quad \{V\mathbf{M}\}=0.$$

Excluding j^2 from these equations, we obtain

$$\{(P+w_0)V\}-\frac{V_1+V_2}{2}\{P\}+\frac{V_1+V_2}{2}\frac{\{H_n^2\}}{8\pi}+\frac{V_1+V_2}{2}\{H_nM_n\}-\{V\mathbf{H}\cdot\mathbf{M}\}=0,$$

where subscripts 1 and 2 denote the values of physical quantities in front of and behind shock, respectively. Using Eqs. (4.16), we transform the last equation into the form

$$(\varepsilon_1-\varepsilon_2)+\frac{P_1+P_2}{2}(V_1-V_2)+\pi(V_1-V_2)(M_{n1}-M_{n2})^2=0, \tag{4.18}$$

where $\varepsilon=w_0V$ is the unit mass energy. Equation (4.18) is the generalization of the Hugoniot adiabat equation for shock waves in neutron matter. It differs from the Hugoniot adiabat equation of the standard hydrodynamics [4] by the additional term

$$\pi(V_1-V_2)(M_{n1}-M_{n2})^2=\frac{1}{16\pi}(V_1-V_2)(H_{n1}-H_{n2})^2. \tag{4.19}$$

In its turn, Eq. (4.18) differs from the adiabat equation [3], because the term $(1/16\pi)(V_1-V_2)(\mathbf{H}_{t1}-\mathbf{H}_{t2})^2$ of the standard magnetohydrodynamics is replaced, as follows from Eq. (4.19), by $(1/16\pi)(V_1-V_2)(H_{n1}-H_{n2})^2$.

It is easy to see that Eq. (4.18) can be rewritten in the form of standard Hugoniot adiabat

$$(\varepsilon_1^*-\varepsilon_2^*)+\frac{P_1^*+P_2^*}{2}(V_1-V_2)=0,$$

with the introduction of new variables ε^* and P^* instead of ε and P:

$$\varepsilon^*=\varepsilon+2\pi M_n^2V,\quad P^*=P+2\pi M_n^2.$$

At last we write down the equations for the discontinuities of the velocity and the square of the normal component of magnetic moment:

$$(v_1-v_2)=\sqrt{(P_2^*-P_1^*)(V_1-V_2)},$$

$$2\pi(V_1-V_2)(M_{n1}-M_{n2})=\frac{(v_1-v_2)^2}{V_1-V_2}-(P_2^*-P_1^*)=0.$$

The magnetic moment discontinuity may move with an arbitrary velocity. The movement of the magnetic moment discontinuity causes the change of magnetic moment of media in space-time, which in its turn leads to the radiation of electromagnetic waves. However, we do not study this problem in this paper.

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